CHAPTER 5

THREE PHASE SYSTEMS

5.1 Advantages of Poly Phase System

As compared to single phase system, a poly phase system has the following merits.

1. Generation of poly phase power is cheaper.
2. Poly phase machine has higher efficiency and higher p.f. compared to a single phase machine.
3. For the same size, the output of a poly phase machine is greater than that of a single phase machine. Hence it is lighter and cheaper.
4. Single phase motors are not self starting where as poly phase motors are self starting.
5. The parallel operation of single phase alternators is not very smooth, where as three phase alternators run in parallel with out any difficulty.

5.2 Production Of Three Phase Voltage

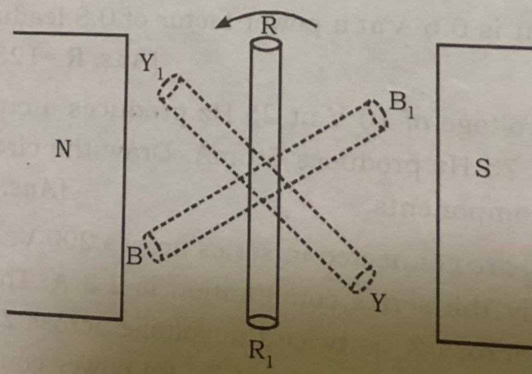

Fig.1 *Generation of three phase e.m.f.*

Figure 1 shows 3 rectangular coils RR_1, YY_1 and BB1 mounted on the same axis but displaced in space from each other by 120^0. Let the 3 coils be rotated anti clockwise with constant angular velocity in a uniform magnetic field between the poles N and S. Let R, Y, B be the start terminals and $R_1Y_1B_1$ be the finish terminals of these coils. Then 'V_{RR_1}' be the voltage induced in the coil RR_1. Similarly V_{YY_1} and V_{BB_1} are the voltages induced in the coils YY_1 and BB_1 respectively. When the complete coil system rotates the emf induced in the three coils are all sinusoidal and equal in magnitude. However, since these coils are displaced 120^0 in space, the emfs V_{RR_1}, V_{YY_1} and V_{BB_1} also have 120^0 phase difference as shown in Fig 3. This system of voltages so obtained are called 3 phase voltages. The instantaneous values of generated emfs in coils RR_1, (phase R), YY_1 (phase Y) and BB_1 (phase B) are given by

$$V_{RR_1} = V_m \sin\theta$$

$$V_{YY_1} = V_m \sin(\theta - 120^0)$$

$$V_{BB_1} = V_m \sin(\theta - 240^0)$$

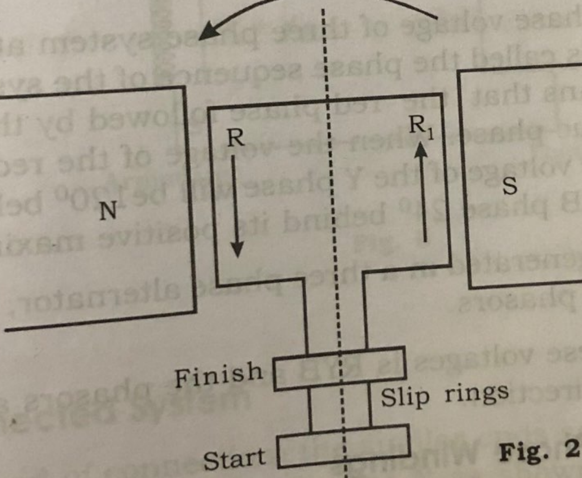

Fig. 2

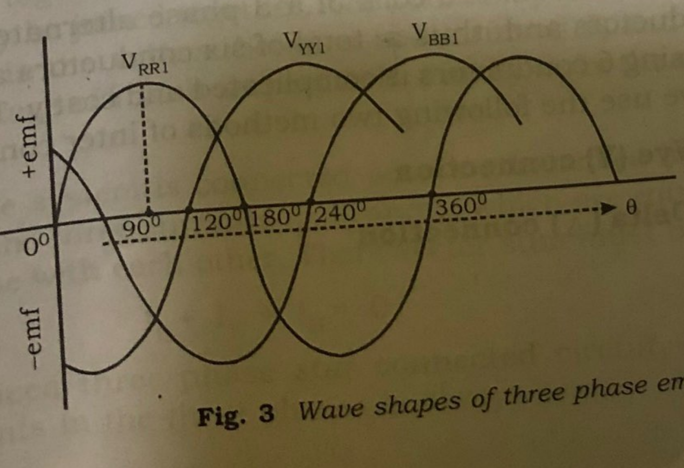

Fig. 3 *Wave shapes of three phase emfs*

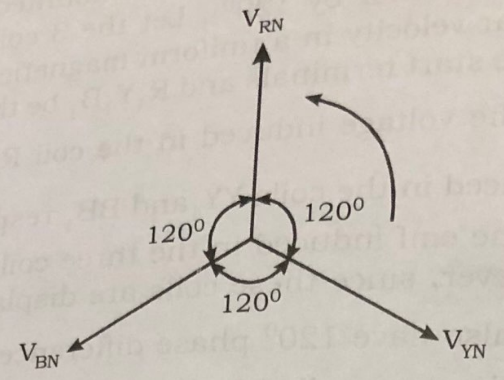

Fig. 4 *Representation of 3 emfs by phasors*

Where V_m is the maximum value of generated emf in each of the coils and θ is the position of the coil RR_1 from its initial position. The rms values (V_{RN}, V_{YN}, V_{BN}) of these three emfs have been represented by phasors in Fig.4.

5.3 Phase Sequence

The order in which the phase voltage of three phase system attain their peak or maximum positive value is called the phase sequence of the system. The phase sequence RYB normally means that the red phase followed by the yellow phase, which is followed by the blue phase. When the voltage of the red phase is at its positive maximum value, the voltage of the Y phase will be 120^0 behind its positive maximum value and that of B phase 24^0 behind its positive maximum value.

Fig. 4 shows the voltages generated in a three phase alternator, in which phase voltages have been shown by phasors.

The phase sequence of these voltages is RYB and the phasors are assumed to rotate in an anti-clockwise direction.

5.4 Connection Of Three Phase Windings

Fig.5 shows three armature coils of a 3 phase alternator. Each phase circuit means two conductors and thus a total of six conductors are needed. This three phase system using 6 conductors is complicated and costly. To reduce the number of conductors we use the following two methods of inter connections.

(i) Star or Wye (Y) connection

(ii) Mesh or Delta (Δ) connection

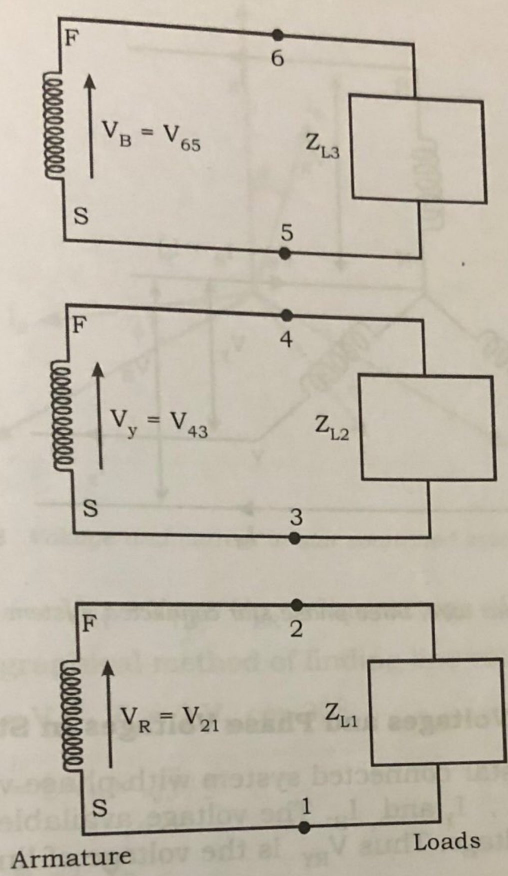

Fig. 5

5.5 Star Connected System

In this method of connection, the similar ends say the start (S) ends of all the three coils are joined together at point N as shown in figure.6. Alternatively all the finish (F) ends may be joined together. The common point 'N' is called neutral point (star point). V_R is the voltage between coil terminal R and the neutral N. Similarly V_Y is the voltage of Y with respect to N. Finally V_B is the voltage of end B with respect to N.

In this three phase system is connected across a balanced load, the neutral wire carries back all the three currents I_R, I_Y and I_B which are equal in magnitude but 120^0 out of phase with each other. There vector sum must be zero. Then,

$$I_R + I_Y + I_B = 0$$

Thus, in a balanced three phase star connected circuit, the sum of the instantaneous currents in the three phases is always zero.

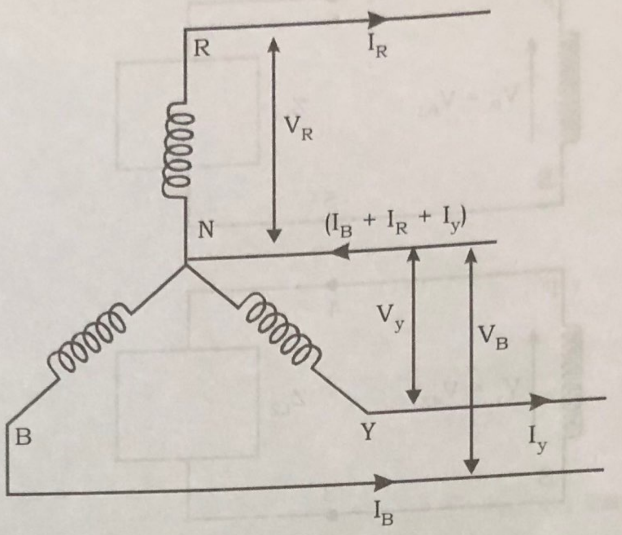

Fig. 6 *Four wire, three phase star connected system*

Relation Between Line Voltages and Phase Voltages in Star Connection

The figure.7 shows a star connected system with phase voltage V_R, V_Y and V_B. The line currents are I_R, I_Y and I_B. The voltage available between any pair of lines is called the line voltage. Thus V_{RY} is the voltage of line R relative to line Y.

The Fig.8 shows the vector diagram for the phase voltages V_R, V_Y and V_B and line currents I_R, I_Y and I_B assuming balanced system.

Thus $V_R = V_Y = V_B =$ phase voltage $= V_P$

The line voltage V_{RY} is the vector difference of V_R and V_Y, i.e., $(V_R - V_Y)$

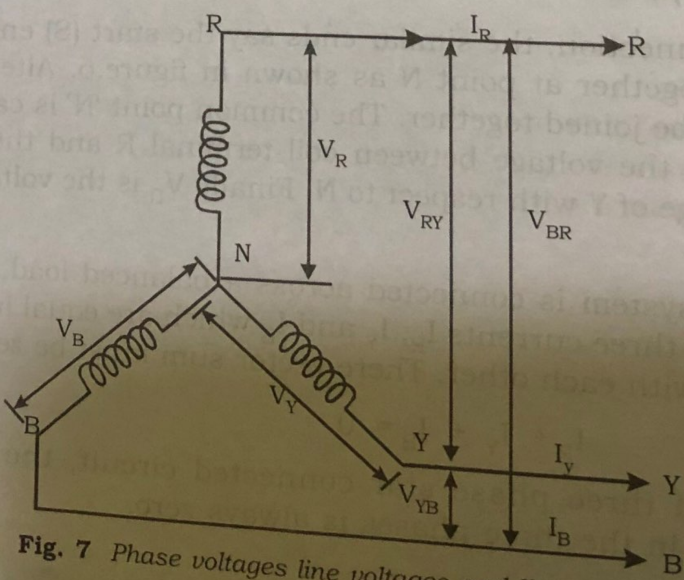

Fig. 7 *Phase voltages line voltages and line current*

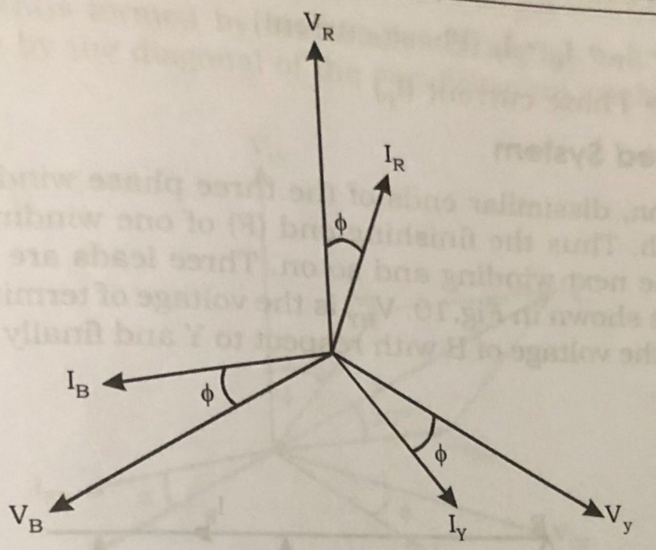

Fig. 8 *Voltage and current in star connected system*

$$V_{RY} = V_{YB} = V_{BR} = \text{line voltage} = V_L$$

Fig. 9 shows the graphical method of finding line voltage V_{RY}.

Line voltage, $V_{RY} = V_R - V_Y = 2\ V_R \cos 30^0$

$$= 2\ V_P \times \sqrt{3}/2$$

$$= \sqrt{3}\ V_P$$

Line voltage = $\sqrt{3}$ × Phase Voltage

Relation between line current and phase current

Fig.7 shows that each line is in series with its individual phase winding. Hence, each line current (I_L) is the same as the phase current (I_P).

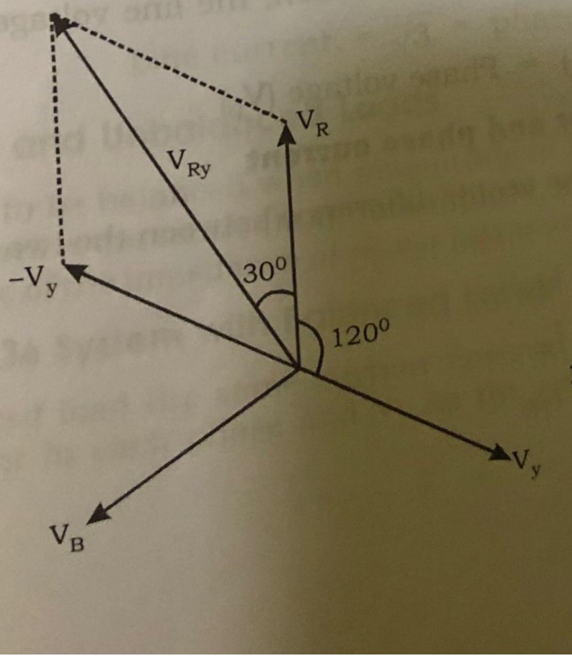

Fig. 9 *Graphical determination of live voltages*

Line current, $I_R = I_Y = I_B = I_P$ (Phase current)

Line current (I_L) = Phase current (I_P)

5.6 Delta Connected System

In delta connection, dissimilar ends of the three phase windings are joined to form the closed mesh. Thus the finishing end (F) of one winding is joined to the starting end (S) of the next winding and so on. Three leads are brought out from the three junctions as shown in Fig. 10. V_{RY} is the voltage of terminal R with respect to Y. Similarly V_{BY} is the voltage of B with respect to Y and finally V_{BR} is the voltage of B with respect to R.

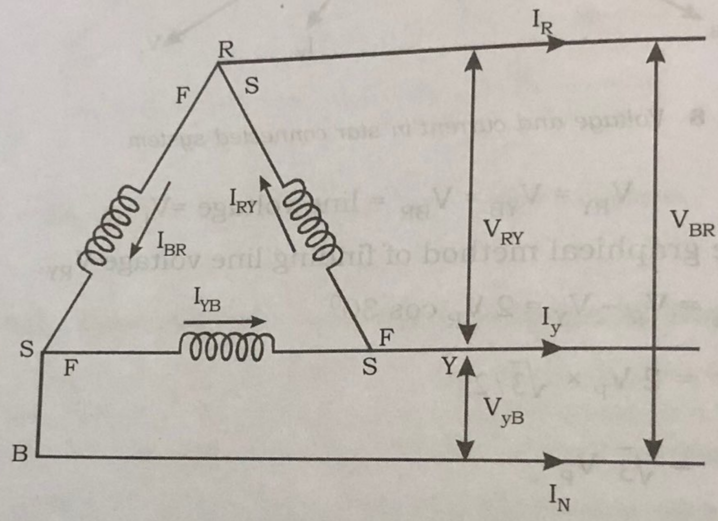

Fig. 10 *Various currents and voltages in delta connected system*

Relation Between Line Voltage and Phase Voltage

Fig. 10 shows that there is only one phase winding completely included between any pair of terminals, Hence in delta connection, the line voltage is equal to the phase voltage.

$$\text{Line voltage } (V_L) = \text{Phase voltage } (V_P)$$

Relation between line current and phase current

The current in each line is the vector difference between the two phase currents involved. Thus

$$I_R = I_{RY} - I_{BR}$$

$$I_Y = I_{YB} - I_{RY}$$

$$I_B = I_{BR} - I_{YB}$$

Current I_R is thus formed by vectorially adding I_{RY} and $-I_{BR}$. Thus then the current I_R is given by the diagonal of the parallelogram as shown in Fig.11.

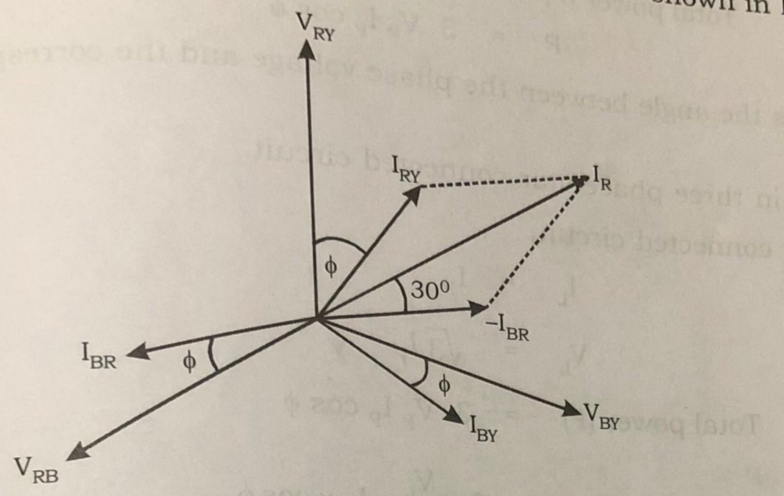

Fig. 11 *Vector addition of phase current to give line current*

The angle between I_{RY} and $-I_{BR}$ is 60^0

$$I_{RY} = I_{YB} = I_{BR} = \text{phase current}(I_P)$$

$$I_R = I_Y = I_B = \text{Line current}(I_L)$$

Line current,

$$I_R = I_{RY} - I_{BR}$$

$$= 2\, I_P \times \cos 30^0$$

$$= 2\, I_P \sqrt{3}/2$$

$$= \sqrt{3}\, I_P$$

$$\text{Line current} = \sqrt{3} \times \text{phase current}$$

5.7 Balanced and Unbalanced Loads

Load is said to be balanced when magnitude of impedance and power factor of load in each phase is the same. On the other hand, the load is said to be unbalanced if the magnitude of the impedance or power factor or both in each phase is different.

5.8 Power in 3ϕ System with Balanced Loads

With balanced load the same current flows in each phase. Let I_P be the rms value of current in each phase and V_P be the rms value of voltage across each phase.

$$\text{Power/phase} = V_P I_P \cos\phi$$

$$\text{Total power (P)} = 3 \times \text{power per phase} = 3 \times V_P I_P \cos\phi$$

$$P = 3\ V_P I_P \cos\phi$$

Where ϕ is the angle between the phase voltage and the corresponding phase current.

(a) Power in three phase star connected circuit.

In star connected circuit,

$$I_L = I_P$$

$$V_L = \sqrt{3}\, I_P$$

$$\text{Total power (P)} = 3\ V_P I_P \cos\phi$$

$$= 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos\phi$$

$$= \sqrt{3}\ V_L I_L \cos\phi$$

Total power $= \sqrt{3} \times$ line voltage $\times$ line current $\times$ power factor

(b) Power in 3 ϕ delta connected circuit

In a delta connected circuit

$$V_L = V_P$$

$$\text{Total power (P)} = 3\ V_P I_P \cos\phi$$

$$= 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi$$

$$= \sqrt{3}\ V_L I_L \cos\phi$$

5.9 Measurement of Power

For measuring the total power supplied to a three phase load, the following methods are applied

(i) The three watt meter method

(ii) The two watt meter method

(iii) The one watt meter method

Of these methods, the two watt meter method is widely adopted in practice.

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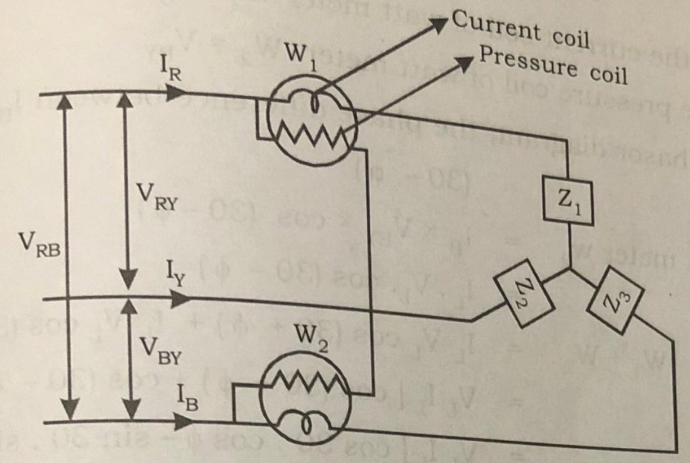
The two watt meter method :



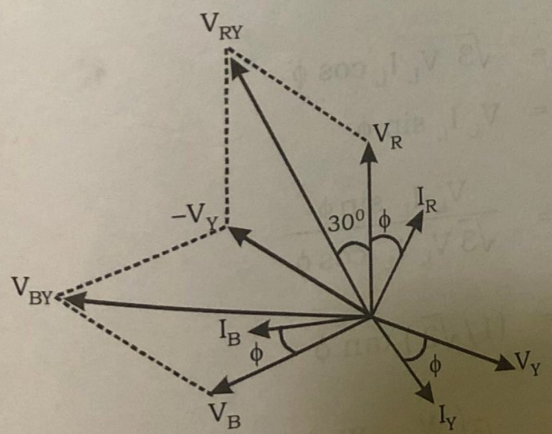
Power in a three phase system, with balanced or unbalanced load can be measured by using two watt meters. The algebraic sum of the two watt meters readily gives three phase power. This can be proved as follows.

Consider a three phase star connected balanced load, supplied from a three phase supply system. Let two watt meters w_1 and w_2 are used to measure the power supplied to the load. Let I_R, I_Y and I_B be the rms values of the currents in the lines. Let V_{RY}, V_{YB}, V_{BR} be the voltages across the lines.

Current through the current coil of watt meter $W_1 = I_R$

Voltage across the pressure coil of watt meter $W_1 = V_{RY}$

Referring to the phasor diagram,



Phase difference between I_R and V_{RY} $= (30 + \phi)$

Reading of watt meter W_1 $= I_R \times V_{RY} \times \cos(30 + \phi)$

$$= I_L \cdot V_L \cdot \cos(30 + \phi)$$

Current through the current coil of watt meter $W_2 = I_B$

Voltage across the pressure coil of watt meter $W_2 = V_{BY}$

Referring to the phasor diagram, the phase difference between I_B and V_{BY}

$$= (30 - \phi)$$

Reading of watt meter w_2

$$= I_B \times V_{BY} \times \cos(30 - \phi)$$

$$= I_L \cdot V_L \cdot \cos(30 - \phi)$$

$$W_1 + W_2 = I_L V_L \cos(30 + \phi) + I_L V_L \cos(30 - \phi)$$

$$= V_L I_L [\cos(30 + \phi) + \cos(30 - \phi)]$$

$$= V_L I_L [\cos 30 \cdot \cos\phi - \sin 30 \cdot \sin\phi + \cos 30 \cos\phi + \sin 30 \cdot \sin\phi]$$

$$= V_L I_L [\cos 30 \cos\phi + \cos 30 \cdot \cos\phi]$$

$$= V_L I_L [2 (\cos 30 \cos\phi)]$$

$$= V_L I_L \left[2 \times \frac{\sqrt{3}}{2} \cos\phi\right]$$

$$= \sqrt{3}\, V_L I_L \cos\phi$$

= Total power input to a balanced load.

Hence the sum of the reading indicated by the two watt meters is equal to the total power drawn by a three phase balanced load.

$$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\phi$$

$$W_1 - W_2 = V_L I_L \sin\phi$$

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi}$$

$$= (1/\sqrt{3}) \tan\phi$$

$$\tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

Effect of power factor on Watt meter readings :

(i) Both the watt meters will indicate the same readings, when the power factor of the load is unity i.e.,

$$\cos\phi = 1$$

$$\phi = 0$$

$$\text{i.e., } W_1 = W_2$$

(ii) When $\phi = 60^0$ i.e., power factor = cos 60 = 0.5 then $W_1 = 0$ and W_2 is positive and measures the entire three phase power.

(iii) When $\phi = 90^0$ i.e., power factor = cos 90^0 = 0. It is seen that $-W_1 = W_2$. i.e., the two readings are equal but are of opposite signs.

$\therefore W_1 + W_2 = 0$, since the power factor is zero.

(iv) Reading of watt meter W_1 is negative for the load power factor less than 0.5 lagging. In such case it is necessary to reverse the connections to either the current or pressure coil, in order to measure the power registered by watt meter W_1. However, the reading thus obtained must be taken as negative, while calculating the total power and the power factor.

PROBLEMS

1. The input power to a three phase motor was measured using two watt meters. The readings were 5.2 kW and −1.7kW, and the line voltage was 415V. Calculate (a) the total active power, (b) the power factor, and (c) the line current.

Solution :

(a) Total power $= W_1 + W_2 = 5.2 - 1.7 = 3.5 \text{ k W}$

(b) Power factor $= \cos\left[\tan^{-1}\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}\right] = \cos\left[\tan^{-1}\sqrt{3}\,\frac{5.2+1.7}{5.2-1.7}\right]$

$= \cos\tan^{-1}[3.415] = \cos(73.68^0) = 0.28$

(c) We know $P = \sqrt{3}\, V_L I_L \cos\phi$

So $I_L = \frac{P}{\sqrt{3} \times V_L \times \cos\phi} = \frac{3.5 \times 1000}{\sqrt{3} \times 415 \times 0.28} = 17.39 \text{ A}$

2. In a two watt meter method to measure power in a three phase circuit, it was found that the two watt meters read 3 kW and 1.5 kW respectively. Determine the total power consumed and the power factor of the balanced three phase circuit.

Solution :

Given : $W_1 = 3 \text{ k W}$, $W_2 = 1.5 \text{ k W}$

The total power consumed $= W_1 + W_2 = 3 + 1.5 = 4.5 \text{ k W}$

The power factor angle is given by

$$\tan\theta = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3} \times \left[\frac{3 - 1.5}{3 + 1.5}\right] = \sqrt{3} \times \frac{1.5}{4.5}$$

$$\therefore \quad \theta = \tan^{-1}\frac{1}{\sqrt{3}} = 30^0$$

Therefore, the power factor is given as

$P f = \cos\theta = \cos 30^0 = \sqrt{3}/2 = 0.866$

3. Each branch of a 3-phase star connected load consists of a coil of resistance 4.2 ohms and reactance 5.6 ohms. The load is supplied at a line voltage of 415 V, 50 Hz. The total power supplied to the load is measured by the two watt meter method. Find the watt meter readings.?

Solution : $V_L = 415$ V, f = 50 Hz

Impedance per phase, $Z_P = R + j X_L = (4.2 + j\,5.6)\ \Omega$

Let W_1 & W_2 be the two meter readings. For a star connected system, we have

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ Volts}$$

$$\text{Phase current, } I_P = \frac{V_P}{Z_P}$$

$$\text{Magnitude of } Z_p = \sqrt{4.2^2 + 5.6^2} = 7\ \Omega$$

$$I_P = \frac{239.6}{7} = 34.229 \text{ A}$$

$$\text{Line current, } I_L = I_P = 34.229 \text{ A}$$

$$\text{Power factor, } \cos\phi = \frac{R}{Z} = \frac{4.2}{7} = 0.6$$

$$\therefore \phi = \cos^{-1}(0.6) = 53.13^0$$

$$\text{Power input, } P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 415 \times 34.229 \times 0.6 = 14.7623 \text{ k W}$$

$$W_1 + W_2 = 14.7623 \text{ k W} \quad \text{......................(1)}$$

$$\tan\phi = \tan 53.13^0 = \sqrt{3} \times \frac{W_1 - W_2}{W_1 + W_2}$$

$$1.333 = \sqrt{3} \times \frac{W_1 - W_2}{14.7623}$$

$$W_1 - W_2 = 11.3637 \text{ k W} \quad \text{......................(2)}$$

From (1) & (2), we get,

$$W_1 = 13.06 \text{ kW and } W_2 = \mathbf{1.699 \text{ k W}}$$

4. Two Watt meters have been used to measure the power input to a 100 kW, 400 V, 3-phase induction motor running at full load. The watt meter readings are 75 kW and 40 k W. Calculate (i) The input to the motor (ii) Power factor of the motor (ii) Line current drawn by the motor and (iv) Efficiency of the motor.

$$W_1 = 75 \text{ k W}, \quad W_2 = 40 \text{ kW}$$

(i) Power input to the motor = $W_1 + W_2$ = 75 + 40 = 115 k W

(ii) $\tan\phi = \sqrt{3} \times \dfrac{W_1 - W_2}{W_1 + W_2} = \sqrt{3} \times \dfrac{75 - 40}{75 + 40} = \sqrt{3} \times \dfrac{35}{115} = 0.527$

$\therefore \quad \phi = \tan^{-1}(0.527) = 27.79^0$

Power factor, $\cos\phi = \cos(27.79) = \mathbf{0.884}$

(iii) Power input to the motor = $\sqrt{3} \cdot V_L I_L \cos\phi$

i.e., $\sqrt{3} \cdot V_L I_L \cos\phi = 115 \times 10^3$

$$\therefore I_L = \frac{115 \times 10^3}{\sqrt{3} \times 400 \times 0.884} = 187.76 \text{ A}$$

Line current, I_L = **187.76 A**

(iv) Output of the motor = 100 k W

Input of the motor = 115 k W

Efficiency of the motor = $\dfrac{\text{Out put}}{\text{In put}} = \dfrac{100}{115} = 0.8695 = \mathbf{86.95\ \%}$

5. A balanced star connected load is supplied from a symmetrical, 3-phase, 400 V, 50 Hz supply system. The current in each phase is 15 A and lags behind its phase voltage b an angle 50^0. Calculate (i) phase voltage (ii) load parameters (iii) total power and (iv) readings of two watt meters, connected in the load circuit to measure the total power.

Solution :

(i) Line voltage, V_L = 400 V

Phase voltage, $V_P = \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 231 \text{ V}$

(ii) Current in each phase, I_P = 15 A

Impedance of the load per phase,

$$Z_P = \frac{V_P}{I_P} = \frac{231}{15} = 15.4\ \Omega$$

$$Z_P = \sqrt{R^2 + X_L^2} \quad \text{.......................(1)}$$

The current in each phase lags behind its voltage by 50^0.

$$\tan 50^0 = \frac{X_L}{R}$$

$$1.191 = \frac{X_L}{R}$$

$$X_L = 1.191\,R$$

Substituting the value of X_L in eqn. (1)

$$Z_P = \sqrt{R^2 + X_L^2}$$

$$15.4 = \sqrt{R^2 + (1.191R)^2}$$

$$(15.4)^2 = R^2 + (1.191)^2R^2$$

$$R^2\,(2.41) = 237.16$$

$$R^2 = \frac{237.16}{2.41} = 98$$

$$R = 9.89\ \Omega$$

Inductive reactance of the load,

$$X_L = 1.191 \times 9.89 = 11.79\ \Omega$$

(iii) Total power $= \sqrt{3}\,.\,V_L I_L \cos\phi = \sqrt{3} \times 400 \times 15 \times \cos 50^0 = 6680$ W

(iv) Total power $= W_1 + W_2$ (Readings of 2 Watt meters)

Thus $W_1 + W_2 = 6680$.......................(2)

$$\tan\phi = \sqrt{3} \times \frac{W_1 - W_2}{W_1 + W_2}$$

$$\tan 50 = \sqrt{3} \times \frac{W_1 - W_2}{6680}$$

$$1.191 = \sqrt{3} \times \frac{W_1 - W_2}{6680}$$

$$W_1 - W_2 = \frac{6680 \times 1.191}{\sqrt{3}}$$

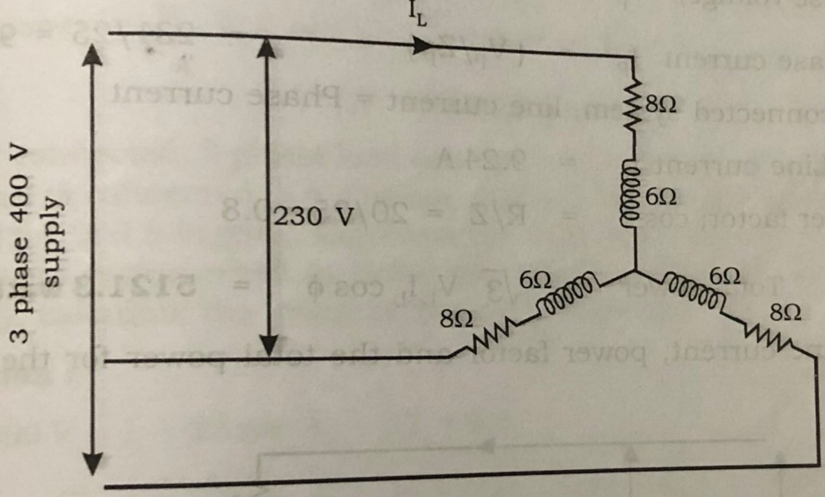
$$W_1 - W_2 = 4596.23 \quad \text{.....................(3)}$$

Solving eqns. (2) and (3),

$$W_1 = 5638.12 \text{ W}$$

$$W_2 = 1041.88 \text{ W}$$

6. A balanced star connected load of (8 + j 6) Ω per phase is connected to a 3 phase, 230 V supply. Find the current, power factor, power and reactive volt-ampere.?



Solution :

Given impedance per phase, Z_P = (8 + j 6)

Line voltage V_L = 230 V

For the star connected system, phase voltage $V_P = V_L / \sqrt{3}$

$= 230/\sqrt{3} = 132.79$ V

Magnitude of $Z_P = \sqrt{(8^2 + 6^2)} = 10\ \Omega$

Phase current $I_P = V_P/Z_P = 132.79/10 = 13.279$ A

Since $I_P = I_L$ Line current I_L = 13.279 A

Power factor, $\cos\phi$ = (R/Z) = (8/10) = 0.8

Total power $P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 230 \times 13.279 \times 0.8 =$ **4232 Watts**

Reactive volt amperes $= \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 230 \times 13.279 \times 0.6$

$=$ **3173.97 VAR**

7. Three impedances each having resistance 20 Ω and an inductive reactance of 15 Ω are connected in star across a 400 V, 3 phase, AC supply. Calculate (a) the line current (b) the power factor (c) total power.

Solution :

$V_L = 400\ V,\quad R = 20\ \Omega,\quad X_L = 15\ \Omega$

Phase impedance, $Z_p = \sqrt{(20^2 + 15^2)} = 25\ \Omega$

Phase voltage, $V_P = (400/\sqrt{3}) = 231\ V$

Phase current $I_P = (V_P/Z_P) = 231/25 = 9.24\ A$

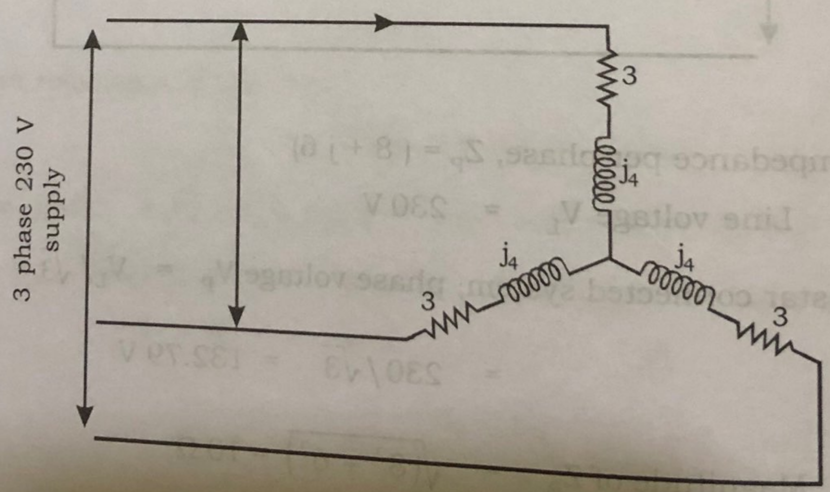
In a star connected system, line current = Phase current

Line current $I_L = 9.24\ A$

Power factor, $\cos\phi = R/Z = 20/25 = 0.8$

Total power $= \sqrt{3}\ V_L I_L \cos\phi =$ **5121.3 watts**

8. Find the line current, power factor and the total power for the given star circuit.



Solution :

Line voltage $V_L = 400\ V$

Phase voltage $V_P = 400/\sqrt{3} = 231\ V = 231\angle 0$ [taken as the reference vector]

Phase impedance $Z_P = (3 + j4) = 5\angle 53.13\ \Omega$

Phase current $I_P = (V_P/Z_P) = (231\angle 0/5\angle 53.13)$

$= 46.2\angle -53.13\ A$

Power factor angle $= 53.13 - 0$

$= 53.13$

Power factor $\cos\phi = \cos(53.13)$

$= 0.6$ lag

Line current, $I_L = I_P$

$= 46.2\ A$

Total power $= \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 46.2 \times 0.6$

$=$ **19204.9 watts**

9. A star connected, 3 phase load consists of three identical impedances. When the load is connected to a 3 phase, 400 volt supply, the line current is 23.09 A and p.f. is 0.8 lagging. Calculate the total power taken by the load.? If the load were reconnected in delta and supplied from the same three phase supply, calculate the current flowing in each line.

Solution :

$V_L = 400\ V,\ I_L = 23.09\ A,\ p.f = 0.8$

Power $= \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 23.09 \times 0.8 = 12797.7$ watts

Since $I_p = I_L$

Line current $I_L = 23.09\ A$

$V_P = V_L/\sqrt{3} = 400/\sqrt{3} = 231\ V$

$Z_P = V_P/I_P = 231/23.09 = 10\ \Omega$

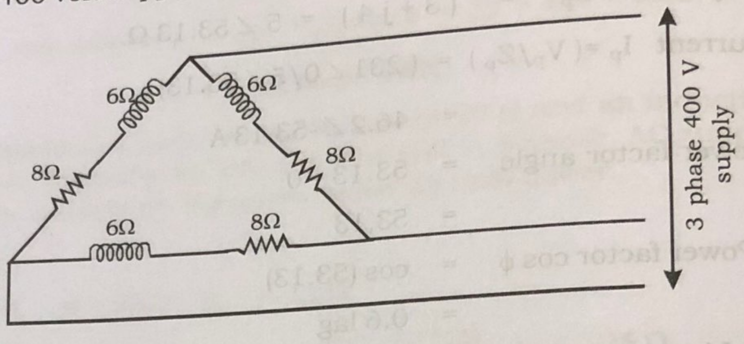
On connecting the load in delta :

$V_p = V_L = 400\ V$

$I_p = V_p/Z_p = 400/10 = 40\ A$

Line current $I_L = \sqrt{3}\ I_p = \sqrt{3} \times 40 = 69.28\ A$

10. A balanced delta connected load of $(8 + j\,6)\ \Omega$ per phase is supplied from 3 phase, 400 volt supply. Find the line current, power factor and total power.



Solution :

Given impedance per phase, $Z_p = (8 + j\,6)$

Line voltage $V_L = 400$ V

For the delta connected system phase voltage $V_P = V_L = 400$ V

Magnitude of $Z_P = \sqrt{(8^2 + 6^2)} = 10\ \Omega$

Phase current $I_P = V_P/Z_P$

$= 400/10 = 40$ A

Line current $I_L = \sqrt{3} \cdot I_P$

$= 40 \times \sqrt{3} = 69.28$ A

Power factor, $\cos\phi = (R/Z_P)$

$= (8/10) = 0.8$

Total power, $P = \sqrt{3}\ V_L I_L \cos\phi$

$= \sqrt{3} \times 400 \times 69.28 \times 0.8$

$=$ **38398 watts**

11. Three similar resistors are connected in star across 400 volts, 3 phase lines. The line current is 10 Amps. If the same resistors are connected in delta across the same supply. Calculate the line current drawn from the supply?

Ans. When the resistors are connected in star,

$I_L = I_{ph} = 10$ A

$V_L = 400$ V

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.9$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{230.9}{10} = 23.09\ \Omega$$

When the resistors are connected in delta, $V_{ph} = V_L = 400$ V

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{23.09} = 17.32\ A$$

$$I_L = \sqrt{3}I_{ph} = \sqrt{3} \times 17.32 = 10\ A$$

12. In a balanced 3-phase circuit, power is measured by two wattmeters. Find the readings of the two wattmeter in the following cases:
(i) the load is 20 kW at unity power factor
(ii) the load is 20 kW at 0.8 power factor
(iii) the load is 20 kW at 0.5 power factor
(iv) the load is 20 kW at 0.25 power factor

Ans. (i) $W_1 = 10$ kW and $W_2 = 10$ kW

(ii) $W_1 + W_2 = 20$ kW ....(1)

$\cos\phi = 0.8$

$\tan\phi = 0.75$

$$\tan\phi = \sqrt{3}\left[\frac{W_1 - W_2}{W_1 + W_2}\right]$$

$$0.75 = \sqrt{3}\left[\frac{W_1 - W_2}{20}\right]$$

$W_1 - W_2 = 8.66$ ....(2)

By solving equations (1) & (2) we get

$W_1 = 14.33$ kW & $W_2 = 5.67$ kW

(iii) $W_1 = 0$ & $W_2 = 20$ kW

(iv) $W_1 + W_2 = 20$ kW ....(1)

$\cos\phi = 0.25$

$\tan\phi = 3.872$

$$\tan\phi = \sqrt{3}\left[\frac{W_1 - W_2}{W_1 + W_2}\right] \quad ....(2)$$

By solving equation (1) and (2) we get

$W_1 = 32.36$ & $W_2 = -12.36$

13. Three similar inductive coils are connected in star to a 3-phase, four wire, 415 V, 50 Hz supply. The line current is 4 A at a p.f. of 0.6 lagging. Calculate the resistance and inductance of one coil?

Ans. Line voltage, $V_L = 415\text{ V}$

Phase voltage, $V_P = \dfrac{415}{\sqrt{3}} = 240\text{ V}$

Power factor $(\cos\phi) = 0.6$

$\therefore \phi = 53.13^0$

Line current, $I_L = 4\text{ A}$

Here, Line current = Phase current

Phase current, $I_P = 4\text{A}$

Impedance per phase, $Z_P = \dfrac{V_P}{I_P} = \dfrac{240}{4} = 60\ \Omega$

$$Z_P = \sqrt{R^2 + X_L^2} \quad \text{............(1)}$$

The current in each phase lags behind its voltage by 53.13^0

$$\tan\phi = \frac{X_L}{R}$$

$$X_L = R \times \tan\phi = R \times \tan 53.13$$

$$= R \times 1.33 = 1.33\text{ R}$$

Substituting the value of X_L in equation (1)

$$Z_P = \sqrt{R^2 + x_L^2}$$

$$60 = \sqrt{R^2 + (1.33R)^2}$$

$$60^2 = R^2 + 1.77\ R^2$$

$$R^2 = 1299.63$$

$$R = 36\ \Omega$$

Inductive reactance of the coil,

$$X_L = 1.33 \times 36 = 48\ \Omega$$

14. A balanced 3-phase delta connected load consists of $10\angle 60^0$ ohms impedance in each phase. 400 volts, 3-phase supply is applied to this circuit. Calculate the current drwan from the supply.

Ans. For a delta connected load,

$$I_L = \sqrt{3}\ I_{Ph} \ \&\ V_{Ph} = V_L;\ V_{Ph} = 400 \text{ Volts},\ Z_{Ph} = 10\angle 60$$

$$I_{Ph} = \frac{400\angle 0}{10\angle 60^0} = 40\angle -60^0$$

$$\therefore I_L = \sqrt{3} \times 40 = 69.28\text{ A}$$

15. A 3-phase delta connected 415 V load has a p.f. of 0.8 lagging and the power consumed by it is 4.35 kW. Calculate the line current drawn from the supply.

Ans. Power consumed = 4.35 kW

Voltage = 415 V

Power factor = 0.8 lagging

For a delta connected load $V_L = V_{Ph}$ and $I_L = \sqrt{3}\, I_{Ph}$

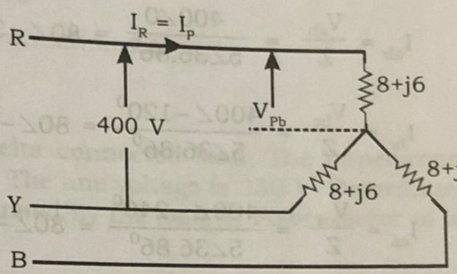
Power, $P = \sqrt{3}\, V_L I_L \cos\phi$

$$I_L = \frac{P}{\sqrt{3} \times V_L \times \cos\phi} = \frac{4.35 \times 10^3}{\sqrt{3} \times 415 \times 0.8} = 7.56 \text{ A}$$

16. A balanced star connected load of 8 + j6 ohms per phase is connected to a three phase 400 V supply. Find the line current, power factor, active power, apparent power and the reactive power.

Ans. $V_L = 400\angle 0^0$

For star connected load, $V_L = \sqrt{3}\, V_{Ph}$



$$V_{Ph} = \frac{400}{\sqrt{3}} = 230.94\angle 0^0$$

$$I_{Ph} = \frac{230.94\angle 0^0}{10\angle 36.86^0} = 23.09\angle -36.86^0 \text{ A}$$

For star connected load, $I_L = I_{Ph}$

Line current, $I_L = 23.09\angle -36.86$ A

Power factor = Cos (36.86^0) = 0.8 lagging

Active power, $P = \sqrt{3} \times 400 \times 23.09 \times 0.8 = 12.79$ kW

Apparent power $= \sqrt{3} V_L I_L = \sqrt{3} \times 400 \times 23.09 = 15.99$ kVA

Reactive power $= \sqrt{3} \times 400 \times 23.09 \times 0.6 = 9.6$ kVAR

17. A 3-phase star connected load has an impedance of (8 + j6) ohms in each phase. The load is connected to a 43-phase 400 V, 50 Hz supply. What will be the wattmeter readings if the power is measured by two wattmeter method?

Ans. $V_L = 400$ V $\quad V_{Ph} = \frac{400}{\sqrt{3}} = 231$ V

$I_{Ph} = I_L$ for star connected load, $I_{Ph} = \frac{231\angle 0^0}{10\angle 36.86^0} = 23.1\angle -36.86^0$

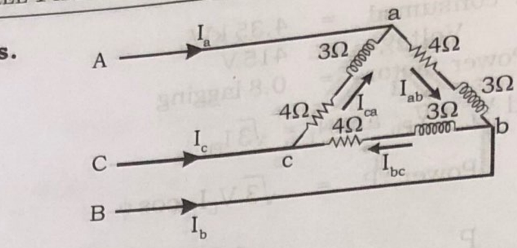
$W_1 = V_{RY} I_R \cos(30+\phi) = 400 \times 23.1 \times \cos 66.86 = 3631.12$ W

$W_2 = V_{BY} I_B \cos(\phi - 30) = 400 \times 23.1 \times \cos 6.86 = 9173.85$ W

$W = W_1 + W_2 = 12.804$ KW

18. Three coils each of resistance 4Ω and inductive reactance 3Ω are connected in Delta across 400V, 50 Hz supply. Find current in each coil, line current, active power and reactive power.

Ans.



$V_P = V_L = 400$ V, $Z_{Ph} = 4 + j3\ \Omega = 5\angle 36.86^0\ \Omega$

Assume ABC as phase sequence

$V_{ab} = 400\angle 0$, $V_{bc} = 400\angle -120^0$, $V_{ca} = 400\angle -240^0$

$$I_{ab} = \frac{V_{ab}}{Z} = \frac{400\angle 0}{5\angle 36.86^0} = 80\angle -36.86^0 \text{ A}$$

$$I_{bc} = \frac{V_{bc}}{Z} = \frac{400\angle -120^0}{5\angle 36.86^0} = 80\angle -156.86^0 \text{ A}$$

$$I_{ca} = \frac{V_{ca}}{Z} = \frac{400\angle -240^0}{5\angle 36.86^0} = 80\angle -276.86^0 \text{ A}$$

The line current are found by applying KCL at nodes a, b and c.

$I_a = I_{ab} - I_{ca} = (80\angle -36.86^0) - (80\angle -276.86^0) = 138.56\angle -66.86^0$ A

$I_b = I_{bc} - I_{ab} = (80\angle -156.86^0) - (80\angle -36.86^0) = 138.56\angle -186.86^0$ A

$I_c = I_{ca} - I_{bc} = (80\angle -276.86^0) - (80\angle -156.86^0) = 138.56\angle -306.86^0$ A

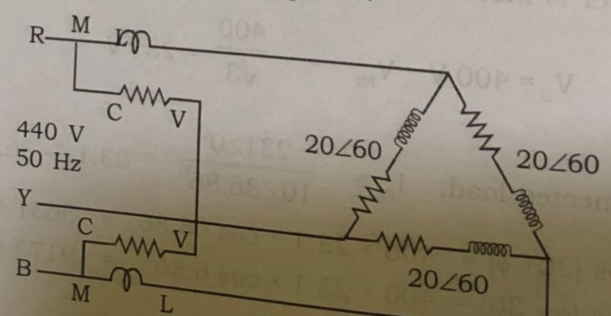
$I_L = I_a = I_b = I_c$

Active power $= \sqrt{3} \times V_L \times I_L \times \cos\phi = \sqrt{3} \times 400 \times 138.56 \times 0.8 = 76.8$ kW

Reactive power $= \sqrt{3} \times V_L \times I_L \times \sin\phi = \sqrt{3} \times 400 \times 138.56 \times 0.6 = 57.6$ kVAR

19. Three identical impedances of 20 < 60^0 ohm connected in delta is fed with 3 - phase, 440v, 50Hz supply. What will be the readings of the two wattmeters connected to measure the total power.

Ans.



For delta connected system

$V_L = V_{ph}$ and $I_L = \sqrt{3}\, I_{ph}$

$I_{ph} = \frac{400\angle 0}{20\angle 60} = 22\angle -60$ A, $\quad I_L = \sqrt{3} \times 22 = 38.1$ A

Total power $= \sqrt{3} \times 440 \times 38.1 \times \cos 60 =$ **14518 Watts**

Total power from the wattmeters $= W_1 + W_2 = 14518$ ....(1)

$\tan\phi = \sqrt{3} \times \frac{W_1 - W_2}{W_1 + W_2}$

$1.732 = \sqrt{3} \times \frac{W_1 - W_2}{14518}$

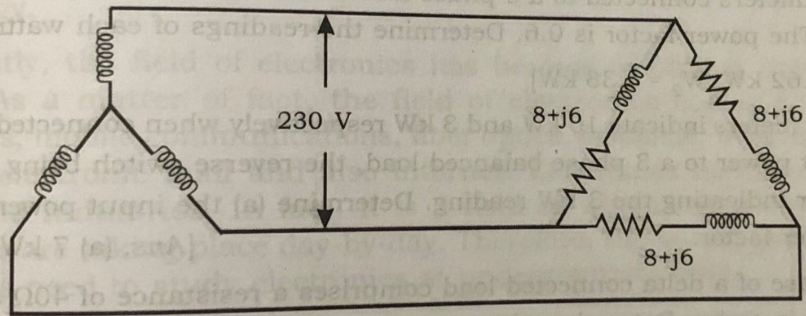
$W_1 - W_2 = 14518$ ....(2)

By solving eqn (1) and eqn (2), we get

$W_2 = 0$ and $W_1 = 14518$ watts

20. A star connected alternator supplies a delta connected load. The impedance of the load branch is (8 + j6) ohm per phase. The line voltage is 230 V. Determine (a) current in the load branch (b) Power consumed by the load (c) Power factor of load (d) Reactive power of the load.

Ans.



Phase current through the load $I_{ph} = \frac{230\angle 0}{8 + j6} = 23\angle -36.86$

Power factor of the load $= \cos 36.86 = 0.8$ lagging

Power consumed by the load $= 3V_{ph}\, I_{ph} \cos\phi = 3 \times 230 \times 23 \times 0.8$

$= 12.7$ kW

Reactive power of the load $= 3\, V_{ph} I_{ph} \sin\phi = 3 \times 230 \times 23 \times 0.6$

$=$ **9.5 kVAR**

PRACTICE PROBLEMS

1. Three loads, each of resistance 50 Ω are connected in star to a 400 V, 3 phase supply. Determine (a) the phase voltage (b) the phase current and (c) the line current. [*Ans.*(a) 231 V (b) 4.62 A (c) 4.62 A]

2. A star connected load consists of three identical coils, each of inductance 159.2 mH and resistance 50Ω. If the supply frequency is 50 Hz and the line current is 3A determine (a) the phase voltage and (b) the line voltage. [*Ans.* (a) 212 V (b) 367 V]

3. Three coils each having resistance 6Ω and inductance L. Henry are connected (a) in star and (b) in delta to a 415 V, 50 Hz, 3 phase supply. If the line current is 30 A, find for each connection the value of L. [*Ans.* (a) 16.78 mH (b) 73.84 mH]

4. Three 24 μF capacitors are connected in star across a 400V, 50 Hz, 3 phase supply. What value of capacitance must be connected in delta in order to take the same line current. [*Ans.* 8μF]

5. The input power to a 3 phase a.c. motor is measured as 5 kW. If the voltage and current to the motor are 400 V and 8.6 A respectively. Determine the power factor of the system. [*Ans.* 0.839]

6. Two wattmeters connected to a 3 phase motor indicate the total power input to be 12 kW. The power factor is 0.6. Determine the readings of each wattmeter. [W_1 = 10.62 kW, W_2 = 1.38 kW]

7. Two watt meters indicate 10 kW and 3 kW respectively when connected to measure the input power to a 3 phase balanced load, the reverse switch being operated on the meter indicating the 3 kW reading. Determine (a) the input power and (b) the load power factor. [*Ans.* (a) 7 kW (b) 0.297]

8. Each phase of a delta connected load comprises a resistance of 40Ω and a 40 μF capacitor in series. Determine when connected to a 1415 V, 50 Hz, 3 phase supply. (a) the phase current (b) the line current (c) the total power dissipated and (d) the kVA rating of the load. [*Ans.* (a) 4.66 A (b) 8.07 A (c) 2.605 kW (d) 5.8 kVA]

9. Three similar coils connected in star take a total power 1.5 kW at a power factor of 0.2 lagging from a three phase, 400 V, 50 Hz supply. Calculate the resistance of the coil. [*Ans.* 7.11Ω]

10. A balanced delta connected load takes line current of 18 A from a 400 V, three phase 50 Hz supply. Calculate the resistance of each leg of the load. [*Ans.*38.49 Ω]